Code No.0427/CORE

FACULTY OF SCIENCE

M.Sc. III Semester Examination, December 2019

Subject: Mathematics

Paper – I: Functional Analysis

Time: 3 Hours | Max. Marks: 80

Note: Answer all questions from Part – A and Part – B. Each question Carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (8x4 = 32 Marks)
(Short Answer Type)

1. State and prove Translation invariance lemma.
2. Let T be a linear operator on a normed space X. If $\dim D(T)=n<\infty$, then prove that $\dim R(T)\leq n$.
3. Prove that the space C[a,b] is not an inner product space.
4. If in an inner product space, $x_n \to x$ and $y_n \to y$, then prove that $<x_n, y_n> \to <x, y>$.
5. Let T be a bounded linear operator on a complex inner product space X and $<Tx, x> = 0$ for all $x \in X$, then prove that T = 0, zero operator.
6. Let T and S be bounded linear operators on a Hilbert space H and α any scalar. Then prove that (i) $(S+T)^*=S^*+T^*$ and (ii) $(\alpha T)^* = \bar{\alpha} T^*$.
7. Let X be a normed space and let $x \neq 0$ be any element of X. Then prove that there exists a bounded linear functional f on X such that $\|f\| = 1$ and $f(x) = \|x\|$.
8. Prove that the normed space X of all polynomials with norm defined by $\|x\| = \max_j \|\alpha j\|$ ($\alpha 0$, $\alpha 1$ ..... the coefficients of x) is not complete.

PART – B (4x12=48 Marks)
(Essay Answer Type)

9. (a) Let $\{x_1,\ldots, x_n\}$ be a linearly independent set of vectors in a normed space X. Then prove that there is a number c > 0 such that for every choice of scalars $\alpha_1, \alpha_2 \ldots\alpha_n$, we have $\|\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_n x_n\| \geq c(|\alpha_1| + \ldots + |\alpha_n|)$ (c>0).

OR

(b) Let $T: D(T) \to Y$ be a bounded linear operator where D(T) lies in a normed space X and Y is a Banach space. Then prove that T has an extension $\overline{T} : \overline{D(T)} \to Y$ where $\overline{T}$ is a bounded linear operator of norm $\|\overline{T}\| = \|T\|$.

10.(a) Prove that the vector space B(X, Y) of all bounded linear operators from a normed space X into a normed space Y is itself a normed space with norm defined by

$$\|T\| = \sup_{\substack{x \in X \\ x \neq 0}} \frac{\|Tx\|}{\|x\|}$$

Also prove that if Y is a Banach space, then B(X, Y) is a Banach space.

OR

(b) State and prove Bessel's inequality.

11.(a) Prove that an ortohonormal set M is a Hilbert space H is total if and only if for all $x \in H$ the Parsevel relation $\sum_k |\langle x, e_k \rangle|^2 = \|x\|^2$ holds.

OR

(b) State and prove Riesz's theorem for linear form.

12.(a) State and prove open mapping lemma.

OR

(b) State and prove closed graph theorem.

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Code No.0430/CORE

FACULTY OF SCIENCE

M.Sc. III Semester Examination, December 2019

Subject: Mathematics

Paper – II: General Measure & Integration

Time: 3 Hours **Max. Marks: 80**

Note: Answer all questions from Part – A and Part – B. Each question Carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (8x4 = 32 Marks)
(Short Answer Type)

1. Let $\langle A_n \rangle$ be a countable collection of measurable sets. Then prove that $\mu\left(\bigcup_{k=1}^{\infty} A_k\right) = \lim_{n\to\infty} \mu\left(\bigcup_{k=1}^{n} A_k\right)$.
2. Prove that every σ = finite measure is saturated.
3. Prove that the union of a countable collection of negative sets is negative.
4. Give an example to show that the Hahn decomposition is not unique.
5. If μ* is the outer measure induced by a measure μ on an algebra $\mathcal{A}$, then prove that each A $\in \mathcal{A}$ *is a* μ*- measurable set.
6. Define product measure.
7. Define inner measure μ_* induced by a measure μ on an algebra.
8. Under usual notations prove that, if $\mu(X) < \infty$, then $\mu_*(E) = \mu(X) - \mu^*(E')$.

PART – B (4x12=48 Marks)
(Essay Answer Type)

9. (a) State and prove generalized Fatou's lemma.

OR

(b) State and prove generalized Lebesgue's convergence theorem.

10. (a) State and prove Jordan decomposition theorem.

OR

(b) State and prove Lebesgue's decomposition theorem.

11. (a) State and Prove Tonelli's theorem.

OR

(b) Let E be a set in $R_{\sigma\delta}$ with $\mu \times v(E) < \infty$. Define g by g(x) = v(Ex). Then prove that g is a measurable function of x and $\int g d\mu = \mu \times v(E)$.

12. (a) If μ* is a Caratheodory outer measure with respect to Γ, then prove that every function in Γ is μ* - measurable.

OR

(b) Let μ be a measure on an algebra $\mathcal{A}$ of subsets of X and E any subset of X. If $\mathcal{B}$ is the algebra generated by $\mathcal{A}$ and E and if $\bar{\mu}$ is any extension of μ to $\mathcal{B}$, then prove that $\mu_*(E) \le \bar{\mu}(E) \le \mu^*(E)$.

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Code No.0433/CORE

FACULTY OF SCIENCE

M.Sc. III Semester Examination, December 2019

Subject: Mathematics/Applied Mathematics

Paper – III: Linear Algebra

Time: 3 Hours **Max. Marks: 80**

Note: Answer all questions from Part – A and Part – B. Each question Carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (8x4 = 32 Marks)
(Short Answer Type)

1. Give an example of a 4X4 matrix for which the minimal polynomial is x^3.
2. Let T: V→V and U: V→V be any two linear operators on a finite dimensional vector space V such that TU=UT. Then show that the null space of U is invariant under T.
3. Let E be a projection of V. Then show that $\beta \epsilon R(E) \Leftrightarrow E\beta = \beta$ (where R(E) is the range of E).
4. Give an example of two 4x4 nilpotent matrices which have the same minimal polynomial but not similar.
5. Let V be a finite dimensional vector space over a field F. and T:V→V be a linear operator and $f(x) \epsilon F[x]$ then show that, for any $\alpha \epsilon V$ $f\,Z(\alpha;T) = Z(f\alpha:T)$.
6. Let T be a linear operator on a finite dimensional vector space V and let $V = W_1 \oplus W_2 \oplus W_3 \oplus ... \oplus W_k$ be the primary decomposition for T, where W_i is the nullspace of $p_i(T)^{r_i}$ and $p = p_1^{r_1}.\, p_2^{r_2}.........p_k^{r_k}$ is the minimal polynomial of T. If W is any subspace of V which is invariant under T, then show that $W = (W \cap W_1) \oplus (W \cap W_2) \oplus ...... \oplus (W \cap W_k)$.
7. State and prove the polarization identity of a bilinear form.
8. Let f be the bilinear form on R^2 defined by $f((x_1, y_1), (x_2, y_2)) = x_1y_1 + x_2y_2$. The find the matrix of f in the base B = {(1, 4), (2, 5)}.

PART – B (4x12=48 Marks)
(Essay Answer Type)

9. (a) Let V be a finite dimensional vector space over the field F and T be a linear operator on V. Then show that T is diagonalizable if and only if the minimal polynomial for T has the form $p(x) = (x-c_1)(x-c_2)......(x-c_k)$ where $c_1, c_2, ...c_k$ are distinct elements of F.

.....2

OR

(b) Let T be the linear operator on R^3 which is represented in the standard ordered basis by the matrix $\begin{bmatrix} 3 & -2 & -2 \\ 1 & 0 & -2 \\ 3 & -3 & -1 \end{bmatrix}$. Is T diagonalizable? If it is diagonalizable, find a basis of R^3 in which T is diagonalizable.

10. (a) Let T be a linear operator on a finite dimensional Vector space V where $V = W_1 \oplus W_2 \oplus \ldots\ldots \oplus W_k$. Also let $E_i(1 \le i \le k)$ are k projections on V. Now show that each subspace W_i is invariant under T if and only if $TE_i = E_iT$ for all i = 1, 2……k.

OR

(b) State and prove the primary decomposition theorem.

11. (a) Let A be a 5x5 matrix with the characteristic polynomial $f(x) = (x-7)^5$. Then find all possible rational canonical forms of A.

OR

(b) Let T be a linear operator on the finite dimensional vector space V. then show that T is semi simple if and only if the minimal polynomial p for T is of the form $p = p_1 . p_2 . p_3 \ldots\ldots p_k$ where $p_1, p_2 \ldots\ldots p_k$ are distinct irreducible polynomials over the scalar field F.

12. (a) (i) Define Skew symmetric bilinear form. Show that every bilinear form can be written uniquely as a sum of a symmetric and skew symmetric bilinear forms.

(ii) Let f be a bilinear form on R^3 defined by

$$f((x_1, x_2, x_3), (y_1, y_2, y_3)) = x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1y_3$$

where (x_1, x_2, x_3), $(y_1, y_2, y_3) \in \mathbf{R}^3$. Now find a basis of $\mathbf{R}^3$ in which f can be represented by block diagonal matrix.

OR

(b) Let V be a finite-dimensional vector space over the field of characteristic zero and let f be a symmetric bilinear form on V. Then show that there is an ordered basis for V in which f is represented by a diagonal matrix.

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Code No.0438/CORE

FACULTY OF SCIENCE

M.Sc. III Semester Examination, December 2019

Subject: Mathematics/Applied Mathematics/Maths with Computer Science

Paper – IV (A): Operations Research

Time: 3 Hours | **Max. Marks: 80**

Note: Answer all questions from Part – A and Part – B. Each question Carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (8x4 = 32 Marks)
(Short Answer Type)

1. Use graphical method to solve the following LPP.

   Max $Z = 2x_1 + 3x_2$ subject to the conditions

   $x_1 + x_2 \leq 1$

   $3x_1 + x_2 \leq 4$

   $x_1, x_2 \geq 0$

2. Construct the dual to the following Primal problem:

   Max $Z = 3x_1 + 5x_2$ subject to the conditions

   $2x_1 + 6x_2 \leq 50$

   $3x_1 + 2x_2 \leq 35$

   $5x_1 - 3x_2, \leq 10$

   $x_2 \leq 20$

   $x1, x2 \geq 0$

3. Formulate the following transportation problem as a LPP.

	I	II	III	a_i
A	1	2	3	4
B	3	1	4	6
b_j	3	3	4	

4. Write a short note on traveling salesman problem.
5. State and establish Bellman's principle of optimality.
6. Write the characteristics of Dynamic programming problem.
7. Briefly explain the three types of floats in Network Analysis.
8. Write the rules for drawing network diagram and explain them.

PART – B (4x12=48 Marks)
(Essay Answer Type)

9. (a) Use the Big-M-Method to solve the following LPP.

   Min $z = 4x_1 + x_2$ subject to the conditions

   $3x_1 + x_2 = 3$

   $4x_1 + 3x_2 \geq 6$

   $x_1 + 2x_2, \leq 3$

   $x_1, x_2 \geq 0$

OR

(b) Use duality to solve the following LPP.

Min $Z = x_1 - x_2$ subject to the conditions

$2x_1 - x_2 \geq 2$

$-x_1 + x_2 \geq 1$

$x_1, x_2 \geq 0$

10.(a) Find the optimal solution of the Transportation problem using MODI method.

	1	2	3	4	a_i
1	8	10	7	6	50
2	12	9	4	7	40
3	9	11	10	8	30
b_j	25	32	40	23	

OR

(b) Solve the following Assignment problem using Hungarian method.

Persons \ Jobs	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

11.(a) Solve the following problem using dynamic programming technique.

Min $Z = y_1^2 + y_2^2 + \ldots\ldots + y_n^2$

Subject to $y_1\, y_2\, y_3 \ldots\ldots y_n = b$

Where $y_1, y_2, \ldots\ldots y_n \geq 0$

OR

(b) Solve the following problems using dynamic.

(i) Min $Z = y_1^2 + y_2^2 + y_3^2$

subject to $y_1 + y_2 + y_3 \geq 15$

$y_1, y_2, y_3 \geq 0$

(ii) Determine the values of u_1, u_2, u_3 so as maximize $u_1\, u_2\, u_3$ subject to $u_1+u_2+u_3 = 10$ and $u_1, u_2, u_3 \geq 0$.

12.(a) The utility data for a network are given below. Determine the total, free, independent floats and identify the critical path.

Activity:	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
Duration:	2	8	10	6	3	3	7	5	2	8

OR

(b) The time estimates (in weeks) for the activities of a PERT network are given below:

Activity	t_o	t_m	t_p
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

(i) Draw the project network and identify all the paths through it.

(ii) Determine the expected project length.

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Code No.0447/CORE

FACULTY OF SCIENCE

M.Sc. III Semester Examination, December 2019

Subject: Mathematics/Applied Mathematics

Paper – V (B): Numerical Analysis

Time: 3 Hours **Max. Marks: 80**

Note: Answer all questions from Part – A and Part – B. Each question Carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (8x4 = 32 Marks)
(Short Answer Type)

1. Explain about Chebyshev method.
2. Show that Newton-Raphson method has second order convergence.
3. Explain about Gauss Jacobbi method.
4. Solve $2x_1 + 2x_2 + x_3 = 1$, $4x_1 + 2x_2 + 3x_3 = 2$, $x_1 + x_2 + x_3 = 3$ by using Gauss-elimination method.
5. Find f(3.0) for the following data.

x	-1	2	4	5
F(x)	-5	13	225	625

6. Find the relation between δ and μ.
7. Derive Trapezoidel Rule by using the method of undetermined coefficients.
8. Find u(0.2) bu using Eulers method for the equation $u' = t^2 + u^2, u(0) = 1$ with $h = 0.1$.

PART – B (4x12=48 Marks)
(Essay Answer Type)

9. (a) Define Rate of convergence and show that Secant method has rate of convergence is 1.618. By using Secant method find a real root of the equation $x\text{-}e^{-x} = 0$ correct to the three decimal places.

OR

(b) Explain about Muller method and using this method perform thre iterations to find a root of the equation $x^3 - 5x + 1 = 0$.

10. (a) Explain about Cholesky method and by using this method solve

$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

OR

(b) Explain about SOR method and perform three iterations of SOR method for the equations $2x_1 - x_2 = 7$, $-x_1 + 2x_2 - x_3 = 1$, $-x_2 + 2x_3 = 1$ with w = 1.1716 and $\overline{x}^{(0)} = \overline{0}$.

11.(a) Find f(3.2) for the following data by using Hermite interpolation formulae.

x	f(x)	f′(x)
3.0	1.09861	0.33333
3.5	1.25276	0.28571
4.0	1.38629	0.25000

OR

(b) Explain about least squares approximation of second degree curve. Find the least square approximation of second degree for the following data.

X	-2	-1	0	1	2
F(x)	15	1	1	3	19

12.(a) Derive Gauss-Legendre two point method. Evaluate $\int_2^3 \frac{\cos 2x}{1+\sin x}dx$ by using Gauss Legendre two point method.

OR

(b) Determine u(t) at t=0.2, 0.4, using Runge-Kutta fourth order method for the equation $u' = \frac{t}{u}$ with u(0) = 1 (h = 0.2) and compare the results with the exact solution and also find error at t = 0.2, 0.4.

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